Growth through Heterogeneous Innovations

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Outline

1. Introduction & Motivation
2. Empirical Evidence
3. Theory framework
4. Quantitative Analysis
5. Conclusion

The Engine of Growth: Why Innovation Matters

The study of innovation is central to understanding long-run economic prosperity.

- Endogenous growth theory establishes that sustained growth is driven by purposeful innovative activities, not exogenous shocks.
- Much of this process is Schumpeterian, characterized by creative destruction: new technologies and firms displace old ones.
- Critically, this process is enacted by heterogeneous firms. Understanding the innovation behavior of **entrants** vs. **incumbents** is key.

Two Enduring Empirical Puzzles

Connecting Theory to Data

Standard models often fail to capture two robust, stylized facts:

Puzzle 1: The Firm Size-Growth Anomaly

- Small firms grow systematically faster than large firms.
- Violates Gibrat's Law (growth is independent of size).

Puzzle 2: The Locus of Radical Innovation

- Source of breakthrough innovations?
- Archetypes: Nimble startup vs. corporate R&D lab.
- Systematic differences in innovation strategies.

Research Questions

This paper bridges theory and evidence by addressing three core questions:

1. Are there systematic differences in the **types of innovation** pursued by large vs. small firms?
2. How do these choices impact firm dynamics and aggregate growth?
3. Can a tractable, quantifiable model explain these empirical patterns?

The Core Idea: A New Margin of Heterogeneity

Internal vs. External Innovation

Internal Innovation

- **Definition:** Improving *existing* product lines.
- **Nature:** Incremental, builds on current knowledge.
- **Key Property:** Incentives scale strongly with firm size.

External Innovation

- **Definition:** Creating *new* product lines.
- **Nature:** Radical, explores new technologies.
- **Key Property:** Incentives scale weakly with firm size.

Central Hypothesis

Innovation strategy (Internal vs. External) is an **endogenous outcome** of firm size.

Contributions

Key Contributions

- A tractable growth model with **endogenous innovation choice**.
- Empirical validation using rich **US firm and patent microdata**.
- Quantitative decomposition of the **drivers of aggregate growth**.

Data Sources

A Look at the U.S. Innovation Landscape

The empirical analysis is grounded in two comprehensive micro-datasets for patenting firms:

- **Longitudinal Business Database (LBD):**
  - U.S. Census Bureau's universe of private sector firms with employees.
  - Key variables: Employment, firm age, entry/exit, industry.
- **NBER Patent Database:**
  - All patents granted by the USPTO, matched to LBD firms.
  - Key variables: Patent counts, citations, technology class.

Fact 1: Negative Size-Growth Relationship

The data confirms a well-known anomaly.

- Small firms exhibit significantly higher average forward growth rates.
- This robustly violates Gibrat's Law.
- **Question:** What economic mechanism drives this systematic pattern?

Figure 1: Average Employment Growth by Firm Size Bin.

Fact 2: Declining Innovation Intensity

A similar negative relationship exists for innovation intensity.

- Intensity is measured as **patents per employee**.
- Smaller firms are substantially more innovation-intensive.
- **Question:** Why do large firms, with vast R&D resources, appear less efficient at producing patents per employee?

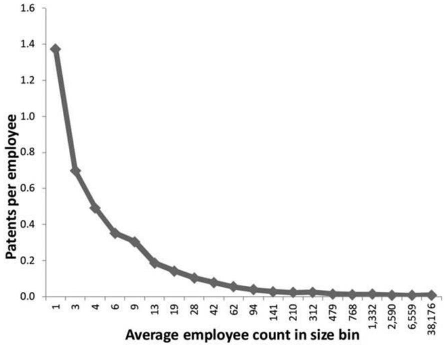

Figure 2: Patents per Employee by Firm Size Bin.

Fact 3a: External Innovations Are Higher-Impact

The classification reveals that external innovations are, on average, of higher quality.

- The distribution for external patents first-order stochastically dominates that of internal patents.
- **Implication:** External innovations receive significantly more citations, indicating higher impact.

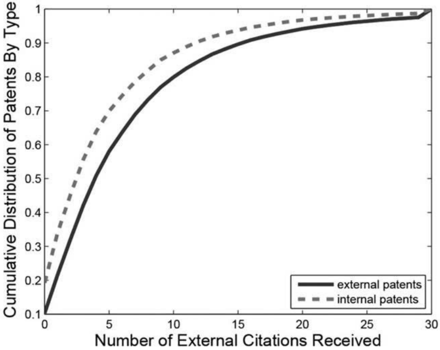

Figure 3: CDF of External Citations Received by Innovation Type.

Fact 3b: Large Firms Shift to Internal Innovation

Regressions of innovation characteristics on firm size (Table 1) reveal that as firms get larger:

- The share of their patents classified as "internal" **increases**.
- The share of patents in the **lowest quality quartile increases**.
- The share of patents in the **highest quality quartile decreases**.

The Puzzle Deepens

Larger firms not only seem less R&D-intensive, but they also appear to endogenously shift their innovation strategy toward safer, lower-impact projects.

Summary of Empirical Regularities

What the Model Must Explain

To summarize, the data reveals an interconnected set of facts:

1. Small firms grow faster.
2. Small firms have higher innovation intensity.
3. Innovation portfolios consist of lower-impact *internal* and higher-impact *external* innovations.
4. Large firms systematically shift towards internal innovation.

The theoretical challenge is to provide a single, coherent mechanism that can jointly rationalize these patterns as endogenous outcomes.

The Economic Environment: Production

A Standard Quality Ladder Setup

The economy consists of a representative household and a production sector.

- A final good Y is produced competitively using labor L and a continuum of intermediate goods $j \in [0, 1]$.

Final Good Production Function

$$Y(t) = \frac{L^{1-\beta}}{1-\beta} \int_0^1 q_j(t) k_j(t)^{\beta} dj \tag{1}$$

- Each intermediate good j is produced by a monopolist. A **firm** is a collection of product lines it monopolizes.
- A firm's size is the sum of the qualities of its products: $Q_f = \sum_{j \in J_f} q_j$.

Intermediate Goods and Monopoly Profits

The producer of the final good has a demand for each intermediate good j given by:

$$p_j = L^{1-\beta} q_j k_j^{\beta-1}$$

The monopolist for good j chooses quantity k_j and quality $q_j$to maximize profit, facing marginal cost w/q_j:

Equilibrium Monopoly Profit

Solving the maximization problem yields an operating profit that is linear in quality:

$$\pi(q_j) = \pi \cdot q_j \tag{2}$$

where π is a constant term determined by primitives (β, L). This linearity ensures that firm value and firm profits scale perfectly with the sum of its product qualities, Q_f.

The R&D Menu: (1) Internal Innovation

Firms can invest to improve their existing products.

Internal R&D

- **Action:** Spend $R_z(z_j, q_j)$ to achieve an arrival rate z_j of innovation.
- **Cost Function:** Cost scales with the quality of the specific product j.

$$R_z(z_j, q_j) = \hat{\chi} z_j^{\hat{\psi}} q_j \tag{3}$$

- **Outcome:** If successful, quality improves by a multiplicative factor λ.

$$q_j \to q_j(1 + \lambda)$$

The returns to this type of R&D scale perfectly with the number of products a firm owns.

The R&D Menu: (2) External Innovation

Firms and entrants can invest to create new product lines.

External R&D

- **Action:** Spend $R_x(x, \bar{q})$ to achieve an arrival rate x of innovation.
- **Cost Function:** Cost scales with the economy's average quality $\bar{q}$.

$$R_x(x, \bar{q}) = \chi x^{\psi} \bar{q} \tag{4}$$

- **Key property:** The cost does **not** depend on the innovating firm's own size. This is the source of the core asymmetry.

The Nature of External Innovations

Successful external innovations can be of two types, creating heterogeneity in quality spillovers:

- With probability θ, the innovation is a **major advance**.
  - It starts a new "technology cluster".
  - The step size is large: $s = \eta\bar{q}$.
- With probability $1 - \theta$, the innovation is a **follow-up improvement**.
  - It improves upon the current technology cluster.
  - The step size decays with the number of prior follow-ups k: $s_k = \eta\alpha^k\bar{q}$, where $\alpha < 1$.

This structure endogenously generates a skewed distribution of innovation quality, consistent with empirical evidence.

Entry and Exit

The economy features a mass of potential entrepreneurs.

Entrant's Problem

Entrepreneurs invest in external R&D to try and enter the market. Their value is determined by the free entry condition, where the value of being an outside entrepreneur, V_0, evolves according to:

$$rV_0 - \dot{V}_0 = \max_{x_e}\{x_e[E_j V(\{q_j + s_j\bar{q}\}) - V_0] - \nu x_e^{\psi}\bar{q}\}$$

- In an equilibrium with positive entry ($x_e > 0$), the expected value of a successful innovation must equal its marginal cost.
- A firm **exits** when it loses its last product line to a competitor.

The Incumbent Firm's Dynamic Problem (HJB)

Synthesizing All Choices and Risks

The value $V(\mathbf{q})$ of an incumbent with product portfolio $\mathbf{q}$ is the solution to:

$$rV(\mathbf{q}) - \dot{V}(\mathbf{q}) = \max_{x,\{z_j\}} \{\dots\}$$

$$\underbrace{\sum_{j\in J_f} \pi q_j}_{\text{Profits}} - \underbrace{\left(\sum_{j\in J_f} R_z(z_j, q_j) + R_x(x, \bar{q})\right)}_{\text{R\&D Costs}} + \underbrace{\sum_{j\in J_f} z_j[V(\mathbf{q}_{+\lambda q_j}) - V(\mathbf{q})]}_{\text{Value Gain from Internal R\&D}} + \underbrace{x[E_j V(\mathbf{q} \cup \{q_j + s_j \bar{q}\}) - V(\mathbf{q})]}_{\text{Value Gain from External R\&D}} - \underbrace{\sum_{j\in J_f} \tau[V(\mathbf{q}) - V(\mathbf{q}_{\setminus j})]}_{\text{Loss from Creative Destruction}}$$

The firm's optimal strategy $\{x^*, z_j^*\}$ emerges from this trade-off.

Balanced Growth Path Equilibrium: Definition

A Balanced Growth Path (BGP) equilibrium is a set of strategies, prices, and aggregate variables such that:

1. **Households** maximize utility.
2. **Final good producers** maximize profits.
3. **Intermediate good producers (Firms)** maximize their value by choosing optimal R&D strategies $\{x^*, z_j^*\}$.
4. The **free entry condition** for entrepreneurs holds.
5. **Markets clear**.
6. Aggregate variables $(Y, C, K, \bar{q})$ grow at a constant rate g.
7. The distributions of firm sizes (μ_n) and innovation step sizes are stationary.

Equilibrium Aggregate Dynamics

In a BGP equilibrium, key aggregate rates are determined endogenously.

Creative Destruction Rate (τ)

The rate at which products are replaced by external innovations is the sum of efforts from all incumbents ($F\bar{x}$) and entrants (x_e):

$$\tau = F\bar{x} + x_e \tag{5}$$

Steady-State Growth Rate (g)

The economy's growth rate is the quality-weighted sum of all successful innovations:

$$g = \tau\bar{s} + \bar{z}\lambda \tag{6}$$

where $\bar{s}$ is the average step size of external innovations.

Equilibrium Firm Size Distribution

In steady state, the flow of firms entering a size class must equal the flow of firms leaving it.

Flow Equations for Firm Size (n products)

$$\text{Inflow into size n} = \text{Outflow from size n}$$
$$F\mu_{n+1}(n+1)\tau + F\mu_{n-1}x = F\mu_n(x+n\tau)$$

Solution

The unique stationary distribution μ_n that solves these equations is a Poisson distribution (truncated at zero):

$$\mu_n = \frac{x_e}{F\bar{x}} \frac{(\bar{x}/\tau)^n}{n!} \quad \text{for } n \geq 1 \tag{7}$$

Key Prediction 1: Firm Growth (Prop. 3)

The model provides a micro-founded explanation for the violation of Gibrat's Law. The expected growth rate of a firm with total quality (size) Q is:

Firm Growth Rate

$$G(Q) = \underbrace{\frac{x(1+\bar{s})\bar{q}}{Q}}_{\text{External R\&D Component}} + \underbrace{\bar{z}\lambda}_{\text{Internal R\&D Component}} - \underbrace{\tau}_{\text{Destruction}} \quad (8)$$

- The contribution from internal R&D is constant with respect to size.
- The contribution from external R&D is a fixed gain, so its percentage contribution decreases as firm size Q increases.

Key Prediction 2: Innovation Intensity (Prop. 4)

Similarly, the model predicts the pattern of declining innovation intensity. The R&D-to-Sales ratio for a firm of size Q is:

R&D Intensity

$$R(Q) = \underbrace{\frac{\beta_c(x)\bar{q}}{\pi Q}}_{\text{External R\&D Intensity}} + \underbrace{\frac{\beta_c(z)}{\pi}}_{\text{Internal R\&D Intensity}} \tag{9}$$

- The intensity of external R&D spending is diluted by the firm's total sales (which are proportional to Q).
- This generates the negative relationship between firm size and innovation intensity (Patents/Employee or R&D/Sales).

Key Prediction 3: Innovation Quality (Prop. 5)

The model explains why large firms produce fewer major innovations.

Probability of a Major Innovation

Conditional on a successful innovation, the probability it is a "major" one for a firm with n products is:

$$M(n) = \frac{\text{Flow of external R\&D}}{\text{Total flow of innovations}} = \frac{x \cdot P(\text{Major})}{x + nz} \tag{10}$$

- The firm's total innovation rate is the sum of its external (x) and total internal (nz) efforts.
- As firm size n increases, the denominator grows, while the numerator is constant.
- Therefore, the share of high-impact innovations is a decreasing function of size.

The Generalized Model for Estimation

Introducing a Scaling Parameter σ

To test the model quantitatively, the authors relax the strict assumption of non-scaling external R&D.

Generalized External Innovation Function

$$X_n = \chi \left[R_x/\bar{q}\right]^{1/\psi} n^{\sigma} \tag{11}$$

- The new parameter σ captures the returns to scale in external R&D.
  - $\sigma = 0$: The baseline model. External R&D is independent of size.
  - $\sigma > 0$: External R&D becomes more effective with firm size.
  - If $\sigma = 1 - \psi$, Gibrat's Law would hold.
- The central empirical goal is to **estimate σ from the data**.

Implications of the Generalized Model

Generalized Value Function (Prop. 6)

Generalizing the model (for $\sigma > 0$) changes the firm's value function. The value of a firm with n products is no longer just the sum of its parts. It has two components:

$$V(q, \bar{q}) = \underbrace{A \sum_{j \in J_f} q_j}_{\text{Value of current products}} + \underbrace{B_n \bar{q}}_{\text{Franchise value of future growth}}$$

- A is the value of owning a single product line.
- B_n is the franchise value of being an n-product firm, representing its capacity for external innovation.
- Unlike the baseline model where B_n is constant, here B_n is an increasing function of n if $\sigma > 0$.

The Key Testable Parameter Combination

What Governs the Size-Intensity Relationship?

The generalized model yields a sharp prediction for innovation intensity $x_n = X_n/n$:

Innovation Intensity (Remark 1)

$$x_n = n^{\xi} f(n) \quad \text{where} \quad \xi = \frac{\psi + \sigma - 1}{1 - \psi}$$

- The relationship between innovation intensity (x_n) and firm size (n) is governed by the exponent ξ.
- Intensity decreases with firm size if and only if $\xi < 0$, which means $\psi + \sigma < 1$.
- The magnitude of the negative size-intensity relationship in the data will identify the value of $\psi + \sigma$. Since ψ (cost curvature) can be taken from literature, this pins down σ.

Identification Strategy: Indirect Inference

The key parameters, especially σ, are estimated by requiring the model to replicate key facts from the US economy.

Objective Function

Find the parameter vector $\Theta = \{\sigma, \chi, \lambda, \dots\}$ that minimizes the distance between data moments and simulated moments:

$$\min_{\Theta} \sum_{i=1}^{7} \frac{|\text{model}_i(\Theta) - \text{data}_i|}{\frac{1}{2}|\text{model}_i(\Theta)| + \frac{1}{2}|\text{data}_i|}$$

Key targeted moments include the size-growth coefficient, R&D intensity, entry rate, and the share of internal patents.

Model Fit: Matching the Moments

Table 5

The calibrated model quantitatively replicates the targeted moments from the US data with high accuracy.

Table 1: Empirical vs. Model-Generated Moments

Moment	**Data**	**Model**
Profitability	.109	.106
R&D Intensity	.041	.042
Internal/External Cite Ratio	.774	.732
Fraction of Internal Patents	.215	.250
Entry Rate	.058	.066
Average Growth Rate	.010	.010
Growth vs. Size Coefficient	-.035	-.035

Estimation Results: Key Parameter Values

Table 6

Table 2: Selected Estimated Model Parameters

Parameter Description	σ (Ext. Scaling)	$\hat{\chi}$ (Cost) (Internal)	χ (Cost) (External)	η (Step) (External)	λ (Step) (Internal)	ν (Entry) (Cost)
Value	0.395	4.066	0.346	0.112	0.051	0.830

- The key finding is $\hat{\sigma} \approx \mathbf{0.4}$. This is significantly different from 0, implying some scaling, but far from the perfect scaling that would restore Gibrat's Law. This indicates decreasing returns in external R&D.
- The quality step-size of major external innovations ($\eta = 0.112$) is more than twice that of internal improvements ($\lambda = 0.051$).

Quantitative Finding 1: The Sources of Growth

The estimated model allows for a decomposition of aggregate productivity growth.

Growth Decomposition (Based on Table 7)

- **New Entrants:** 25.7%
  - Confirms the importance of Schumpeterian creative destruction.
- **Incumbents - Internal Innovation:** 19.8%
  - Incremental improvements from established firms.
- **Incumbents - External Innovation:** 54.5%
  - This is the single **largest driver of growth**: existing firms expanding into new activities.

Quantitative Finding 2: Innovation Impact

The estimated parameters also allow for a direct comparison of innovation quality.

- The average step size associated with external innovations is $\bar{s} = 0.069$.
- The step size of internal innovation is $\lambda = 0.051$.
- This implies that an average external innovation has a 35% larger impact on quality than an average internal innovation.
- This finding provides quantitative backing for the idea that the shift in strategy by large firms is indeed a shift toward lower-impact R&D.

Model Validation: Untargeted Moments

The Strongest Test of a Theory

The model was estimated to match *average linear* relationships. Can it predict the *full non-linear patterns* across the firm size distribution?

Yes.

The model performs remarkably well in matching these untargeted moments (Table 8). It correctly predicts the direction and rough magnitude of change for:

- Slower growth for larger quintiles.
- Lower patents per employee for larger quintiles.
- A higher share of internal patents for larger quintiles.
- A lower share of top-quality patents for larger quintiles.

This strong out-of-sample performance validates the model's core mechanism.

Summary of Contributions

- **Theoretically,** the paper develops a tractable growth model where a single, intuitive mechanism—the differential scaling of internal vs. external R&D costs—endogenously generates a rich set of empirically observed firm dynamics.

- **Empirically,** it provides new, detailed evidence from US microdata on how firms' innovation portfolios systematically change with size, linking this to differences in innovation impact.

- **Quantitatively,** it successfully estimates the model, showing the mechanism is not just qualitatively consistent but quantitatively relevant. The key finding of decreasing returns to external R&D ($\sigma \approx 0.4$) is robust.

Conclusion and Takeaways

Key Finding

Incumbent firms expanding into new product lines (external innovation) is the single largest source of aggregate productivity growth in the US, accounting for over half of the total.

Main Takeaway

The heterogeneity in *how* firms innovate is a critical, and now quantifiable, component for understanding firm dynamics and the micro-foundations of economic growth. The choice between "doing current things better" and "doing new things" is a key margin of adjustment that links firm size, firm dynamics, and macroeconomic performance.